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Analytical Model of Transmission Line Metamaterial with Asymmetricaly Coupled Split-Ring Resonators

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Abstract – **Apart from numerical simulations, some type of analytical model for studying metamaterials is desirable for at least two reasons: easier design and optimization, and, more importantly, improved understanding and intuition about the underlying physics. For transmission line metamaterials, equivalent circuit models were frequently used for this purpose, but they themselves can easily become quite complicated, especially in case of multiple coupled split-ring resonators (SRRs), possibly involving both electric and magnetic coupling. Here we present simple model which starts from coupled linear harmonic oscillators, and then approximates the transmission through the structure. This model is able to provide us with qualitative description of resonance-related phenomena, such as left-handed bands and metamaterial equivalent of electromagnetically induced transparency (EIT).**

I. INTRODUCTION

Metamaterials represent artificial composites, consisting of sub-wavelength resonators, which can exhibit a range of unusual and interesting phenomena [1]. Alongside with three- or two-dimensional volumetric metamaterials, a similar approach can be applied for one-dimensional guided waves, in which case we speak about transmission line metamaterials (TL MM) [2]-[3]. Most concepts, like left-handed transmission bands with negative phase velocity [3], or classical equivalents of electromagnetically induced transparency (EIT) [4], can be implemented in guided-wave structures.

Considering the means for analyzing and studying the TL MMs, we are primarily interested in transmission through the structure. To this end, one usually resorts to 3D electromagnetic simulation as a first step. However, this is often unsatisfactory as simulations can be time-consuming (arguably, this is less and less an issue with the advent of computing power), and, more importantly, this approach does not offer any insight into the underlying physical phenomena. Therefore, a need for a simplified model emerges, and a natural model for TL MMs, already considered by various authors, is an equivalent electrical circuit [3].

Despite its many benefits, namely, the ability to accurately approximate the desired TL MM in a wide range with negligible computation time, there are some drawbacks. For instance, due to the lumped nature of the equivalent circuit, these models generate errors towards at frequencies. Also, when dealing with multiple sub-wavelength resonators, such as split-rings (SRRs) which are mutually coupled, and sometimes exhibit cross-coupling effects, the equivalent circuits can become quite involved, and therefore also quite unintuitive. We should also note that coupled SRRs and related structures have been analyzed in the frame of coupled harmonic oscillators, for instance, when describing EIT analogues in metamaterials [4], but these models rarely account for the coupling with the external excitation, which is needed to obtain the transmission through the structure.

In this paper we propose a model which initially considers electric and magnetic polarizabilities of the constituent microresonators. Then, a simplified approximate expression for transmission is derived. It will be shown that this simple model can qualitatively describe phenomena such as left handed transmission band in the presence of SRR and via, as well as resonance splitting and related transmission zeros in case of coupled SRRs.

II. MODELLING SINGLE SRR COUPLED WITH TRANSMISSION LINE

An example of TL MM consisting of microstrip line loaded with SRR is depicted in Fig. 1a, and the equivalent circuit which can approximate it is shown in Fig. 1b [5]. Analysis shows that the series branch (i.e. coil coupled to the SRR) can be replaced with an equivalent coil:

$$L' = L(1+\kappa),\ \text{where}\ \kappa = \frac{k_m^2\omega^2}{\omega_0^2-\omega^2},\ \omega_0^2 = \frac{1}{L_S C_S}; \tag{1}$$

in this way, κ can be thought of as equivalent magnetic susceptibility. Similarly, if grounding via is present, or SRR is electrically coupled, equivalent capacitance $C' = C(1+\kappa_e)$ can be defined for the shunt branch, where κ_e is equivalent electric susceptibility.

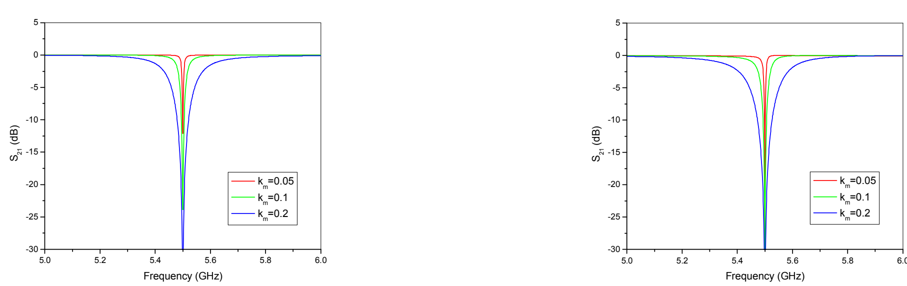
(a) (b)

Fig. 1. (a) SRR coupled with microstrip line (b) equivalent circuit.

Now, we have an equivalent Π-network with series inductance *L'* and shunt capacitance *C'* (we note that in this way cross-coupling effects are not taken into account, but symmetric structures do not exhibit them, and for asymmetric structures this can be considered as first-order approximation). For such network, *S*-parameters can be straightforwardly obtained. After disregarding terms proportional to ω^2, which we consider justified at the frequency range of interest, S_{21} (transmission) reads:

$$S_{21} = \frac{1}{1+\frac{j\omega}{2}(L'-C')} = \frac{1}{1+j\omega\beta(\kappa-\kappa_e)},\ \text{where}\ \beta = \frac{\sqrt{LC}}{2}. \tag{2}$$

Equation (2) gives us simple and intuitive representation of the transmission through the structure. For instance, if there is only magnetic susceptibility given by (1), away from the resonance $S_{21} \approx 1$, and at the resonance $\kappa \to \infty$, so denominator goes to infinity, leading to zero transmission, which is a known behavior structure in Fig. 1a. Equation (2) also shows how electric and magnetic susceptibilities, if equal in magnitude, will cancel out each other, leading to maximal transmission, which can be used as an alternative explanation of left-handed pass-band in TL MM consisting of SRRs and via. In Fig. 2 we show comparison between transmission calculated using our approximate formula and equivalent circuit, demonstrating that it can capture resonance width dependence on coupling strength with high degree of fidelity.



(a) (b)

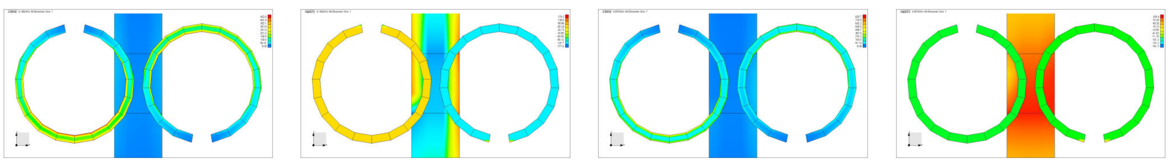
Fig. 2. Calculated logarithmic magnitude of transmission through the microstrip line coupled with single SRR using: (a) equation (2); (b) equivalent circuit in Fig. 1b.

III. Modeling two mutually coupled SRRs

Coupled resonators provide a foundation for many interesting effects, one of which is a classical analog of EIT, leading to high dispersion and extremely low group velocity, which is of both theoretical and practical interest. First step is to consider two magnetically coupled SRRs without transmission line. Then we can write two coupled equations for currents using Kirchhoff's voltage law. To uncouple these equations, we can write currents in terms of symmetric and anti-symmetric normal mode, i.e. $I_{\pm} = I_1 \pm I_2$. Then the equations will split in two independent oscillator equations, with resonance frequencies corresponding to normal modes:

$$\omega_{\pm} = \frac{\omega_0}{\sqrt{1 \pm k_{m2}}}, \tag{3}$$

where k_{m2} is the mutual coupling coefficient. The important point about equation (3) is that, if the SRRs are placed symmetrically in respect to the transmission line, the anti-symmetric mode cannot be excited, in other words it is so-called "dark" mode, and we will have only one resonance in transmission. However, if symmetry is broken in some way, both resonances will appear. To demonstrate this, we simulated two anti-symmetric circular SRRs above the transmission line, and obtained two resonances in transmission, whose current distribution is shown in Fig. 3. From the magnitudes we conclude that the rings are approximately equally excited, and from the phases we can clearly identify symmetric (same phase) and anti-symmetric mode (opposite phase).



(a) (b) (c) (d)

Fig. 3. Current distribution at first resonance: (a) magnitude (b) phase; at second resonance: (c) magnitude (d) phase.

VI. Conclusion

We have presented a simple model for transmission, based on electric and magnetic susceptibilities, which is able to qualitatively reproduce resonance-related phenomena, such as stop-bands and left-handed pass-bands. Further work is planned to explore the coupling of two asymmetric SRRs with the line in detail.

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